(8) $p^2x + q^2y = z$ — (1)

Sol^n:→ Here $f(x,y,z,p,q) = p^2x + q^2y - z = 0$

∵ Charpits. A.E is

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}}$$

i.e $$\frac{dp}{p^2 - p} = \frac{dq}{-q + q^2} = \frac{dz}{-2(p^2x + q y^2)} = \frac{dx}{-2px} = \frac{dy}{-2qy}$$

from which, we have

$$\therefore \frac{p^2 dx + 2pxq}{p^2 x} = \frac{q^2 dy + 2qy\,dy}{q^2 y}$$

integrating we have.

$$\log p^2 x = \log q^2 y + \log a$$

$$\therefore p^2 x = a q^2 y \quad — (2)$$

From (1) & (2)

$$\because p^2 x + q^2 y = z$$

$$\therefore a q^2 y + q^2 y = z$$

$$\therefore q = \left\{\frac{z}{(1+a)y}\right\}^{1/2}$$

$$\& \; p = \left\{\frac{az}{(1+a)x}\right\}^{1/2}$$

Putting $dz = p\,dx + q\,dy$

$$dz = \left\{\frac{az}{(1+a)x}\right\}^{1/2} dx + \left\{\frac{z}{(1+a)y}\right\}^{1/2} dy$$

or, $\sqrt{1+a}\,\frac{dz}{\sqrt{z}} = \sqrt{a}\,\frac{dx}{\sqrt{x}} + \frac{dy}{\sqrt{y}}$

integrating

$\sqrt{(1+a)z} = \sqrt{ax} + \sqrt{y} + b$

(8) $px + qy = pq$

sol^n:→ let $f(x,y,z,p,q) = px + qy - pq = 0$ —(1)

∴ A.E is

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}}$$

or,

$$\frac{dp}{p} = \frac{dq}{q} = \frac{dz}{-p(x-q) - q(y-p)} = \frac{dx}{-(x-q)}$$

$$= \frac{dy}{-(y-p)} = \frac{\partial F}{0}$$

taking 1st two

$$\frac{dp}{p} = \frac{dq}{q}$$

$\Rightarrow \log p = \log q + \log a$

$p = aq$ —(2)

from (1) & (2)

$px - qy - pq = 0$

$aqx + qy = aq^2$

$\Rightarrow q = \frac{y + ax}{a}$

or, $p = aq = y + ax$

$\because dz = p\,dx + q\,dy$

$\therefore dz = (y + ax)\,dx + \left(\frac{y+ax}{a}\right) dy$

$\therefore a\,dz = (y + ax)(dy + a\,dx)$

integrating

$$az = \frac{1}{2}(y + ax)^2 + b$$

(Q) Solve $(p^2 + q^2)\,y = qz$

Sol^n)- Here $f(x, y, z, p, q) = (p^2+q^2)y - qz = 0$ —(1)

$\because$ Charpits. A.E is

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}}$$

or,

$$\frac{dp}{-pq} = \frac{dq}{(p^2+q^2) - q^2} = \frac{dz}{-2p^2y - 2q^2y + qz} = \frac{dx}{-2py}$$

$$= \frac{dy}{-2qy + z}$$

taking 1st two weget

$$\frac{dp}{-\cancel{p}q} = \frac{dq}{pz}$$

or, $p\,dp + q\,dq = 0$

$\Rightarrow p^2 + q^2 = a^2 \quad (\text{say}) \quad \text{— (2)}$

From (1) & (2) we get

$\because (p^2+q^2)y = qz$

$\Rightarrow a^2 y = qz \quad \Rightarrow q = \frac{a^2 y}{z}$

or, $p = \sqrt{a^2 - q^2}$

$= \sqrt{a^2 - \frac{a^4 y^2}{z^2}} = \frac{a}{z}\sqrt{z^2 - a^2 y^2}$

$\because dz = p\,dx + q\,dy$

$\therefore dz = \frac{a}{z}\sqrt{z^2 - a^2y^2}\,dx + \frac{a^2 y}{z}\,dy$

or, $z\,dz = a\sqrt{z^2 - a^2 y}\,dx + a^2 y\,dy$

$\Rightarrow \frac{z\,dz - a^2 y\,dy}{\sqrt{z^2 - a^2 y}} = a\,dx$

putting $z^2 - a^2 y^2 = t^2$

$(\cancel{2z\,dz = 2t\,dt})$

$\therefore dt = a\,dx$

$\therefore t = \sqrt{z^2 - a^2 y} = ax + b$

or $z^2 = a^2 y + (ax+b)^2$

— (1)

$= \frac{dy}{-\frac{\partial f}{\partial q}}$

y

(8) solve $z z x - p x^2 - 2 q x y + p q = 0$

Solⁿ:- Here

$$f(x, y, z, p, q) = z z x - p x^2 - 2 q x y + p q = 0 \quad -(1)$$

∴ Charpits A.E is

$$\frac{\partial p}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{\partial q}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}}$$

$$= \frac{dy}{-\frac{\partial f}{\partial q}}$$

or,

$$\frac{dp}{2z - 2qy} = \frac{dq}{0} = \frac{dz}{px^2 - pq + 2xyq - pq}$$

$$= \frac{dx}{x^2 - q} = \frac{dy}{2xz - p} = \frac{\partial F}{0}$$

∴ $dq = 0$ or, $q = a$

Putting these value in (1)

$$P(x^2 - a) = 2x(z - ay)$$

$$p = \frac{2x(z - ay)}{x^2 - a}$$

∴ $dz = p\,dx + q\,dy$

$$\therefore dz = \frac{2x(z - ay)}{x^2 - a}\,dx + a\,dy$$

$$\frac{dz - a\,dy}{z - ay} = \frac{2x}{x^2 - a}\,dx$$

integrating weget

$\log(z - ay) = \log(x^2 - a^2) + \log b$

$\therefore\ z - ay = b(x^2 - a^2)$

or, $z = ay + b(x^2 - a)$